PHYSICAL REVIEW B VOLUME 46, NUMBER 9 1 SEPTEMBER 1992-I

Gravitational effects on the magnetic attenuation of superconductors

Ning Li and D. G. Torr
Center for Space Plasma and Aeronomic Research, Research Institute, University of Alabama in Huntsville, Huntsville, Alabama 35899
(Received 12 March 1992)

It is found that when gravitational effects are taken into account, the two historical independent traditional hallmarks of ideal superconductors—perfect conductivity (σ) and perfect diamagnetism characterized by zero permeability (μ)—are actually related to each other in terms of a theoretical σ-μ model developed in this paper. Our result suggests a possible importance of gravitational effects in understanding superconductivity.

I. INTRODUCTION

Important space and other possible gravitational experiments are crucially dependent on the use of a superconductor shielding system.[1-3] Experimental efforts to realize idealized near-zero magnetic fields inside superconducting shields, such as by using nested and mechanical expandable superconducting shields, have yielded remnant internal magnetic fields of the order of 10^{-12} t.[4] The attenuation of magnetic fields by superconductors, which is a subject of fundamental importance that as yet has not been satisfactorily explained, has recently become critically important to understanding the interaction between gravitation and superconductivity. We have recently demonstrated that a residual magnetic field, given by Eq. (31) in Ref. 5 as

$$\mathbf{B}(z) \approx -\frac{m^2\mu_g}{q^2\mu}\mathbf{B}_0 , \tag{1}$$

will exist within a superconductor when gravitational effects are considered, where $\mathbf{B}_0$ is the external magnetic field and m and q are the mass and charge of a Cooper pair. Although gravitational effects are usually very weak, they may play a major role when other kinds of forces are nulled, as in the case of superconductivity. Equation (1) indicates that the residual magnetic field is caused by gravitational effects, since the magnetic attenuation coefficient

$$\beta = \frac{m^2\mu_g}{q^2\mu} \tag{2}$$

is entirely determined by the ratio of the gravitomagnetic and magnetic permeabilities μ_g and μ of superconductors. Thus the essence of the problem of magnetic attenuation of superconductors reduces to the determination of values of μ and μ_g for superconductors. This topic forms the substance of the rest of the paper, and a theoretical model relating σ and μ is developed in order to predict the effects of gravitation.

II. CURRENT DENSITIES

In our approach a superconductor is treated as a many-particle system. In the presence of applied electromagnetic and gravitoelectromagnetic potentials $\mathbf{A}$, $\mathbf{\Phi}$, $\mathbf{A}_g, \mathbf{\Phi}_g$, the Lagrangian can be written as

$$L = \sum_j \tfrac{1}{2} m_j v_j^2 - \sum_j q_j(\mathbf{\Phi} - \mathbf{v}_j \cdot \mathbf{A}) - \sum_j m_j(\mathbf{\Phi}_g - \mathbf{v}_j \cdot \mathbf{A}_g) . \tag{3}$$

The canonical momentum for the jth particle computed from the Lagrangian is related to its velocity by

$$-i\hbar\nabla_j = m_j\mathbf{v}_j + q_j\,\mathbf{A} + m_j\,\mathbf{A}_g . \tag{4}$$

When this generalized momentum operator acts on the superconducting condensate wave function $\psi\psi^*$, it yields the macroscopic averaged total charge- and mass-current densities $\mathbf{j}_e^s$ and $\mathbf{j}_m^s$, respectively:

$$\mathbf{j}_e^s = \sum_j \frac{q_j\hbar}{2im_j}(\psi^*\nabla_j\psi - \psi\nabla_j\psi^*) - \sum_j \frac{q_j^2}{m_j}\left[\mathbf{A} + \frac{m_j}{q_j}\mathbf{A}_g\right]\psi^*\psi , \tag{5}$$

$$\mathbf{j}_m^s = \sum_j \frac{\hbar}{2i}(\psi^*\nabla_j\psi - \psi\nabla_j\psi^*) - \sum_j q_j\left[\mathbf{A} + \frac{m_j}{q_j}\mathbf{A}_g\right]\psi^*\psi . \tag{6}$$

Equation (5) is the usual quantum-mechanical equation for a many-particle system,[6] where each particle has charge q_j and mass m_j; ψ is the superconducting order parameter with a phase $\phi, \psi = |\psi|e^{i\phi}$, which forms a superconducting condensate coherent wave such that the local density of superconducting electrons is given by $n^s = |\psi|^2$. Equation (6) is the gravitational analog of Eq. (5), which must apply since the charge and mass belong to a common carrier. For a superconductor one need only consider two kinds of charged particles: the Cooper pairs and lattice ions.

With this information and the Maxwell equations

$$\frac{1}{\mu_0}\nabla\times\nabla\times\mathbf{A}=\mathbf{j}^s_{e,f}+\nabla\times\mathcal{M}+\frac{\partial\mathbf{P}}{\partial t}-\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2}\ , \tag{7}$$

$$\frac{-1}{\mu_{g,0}}\nabla\times\nabla\times\mathbf{A}_g=\mathbf{j}^s_{m,f}+\nabla\times\frac{1}{2}\mathcal{L}+\varepsilon_{g,0}\frac{\partial^2\mathbf{A}_g}{\partial t^2}\ , \tag{8}$$

which explicitly show all current sources, μ and μ_g assume their free-space values. With this approach we are able to interpret the following two features of the macroscopic equations for the vector potential $\mathbf{A}$ and its gravitational counterpart $\mathbf{A}_g$ as it pertains to superconductivity.

(i) BCS theory shows that the superconducting electrons in the Fermi-sea *ground state* are expected to result in the formation of electron pairs with *zero angular momentum and spin*.[7] Those so-called Cooper pairs comprise the superconducting current carriers in a superconductor. We can therefore identify the macroscopic average of the "free" superconducting charge and mass currents

$$\mathbf{j}^s_{e,f}=-Q\left[\mathbf{A}+\frac{m}{q}\mathbf{A}_g\right] \tag{9}$$

and

$$\mathbf{j}^s_{m,f}=-Q\frac{m}{q}\left[\mathbf{A}+\frac{m}{q}\mathbf{A}_g\right] \tag{10}$$

with that carried by the Cooper pairs, where the Cooper-pair mass $m=2m_e$, the charge $q=-2e$, and

$$Q=\frac{q^2}{m}|\psi|^2 \tag{11}$$

is the kernal function. The Cooper pairs can move freely, and since their motion is driven entirely by the applied fields, we refer to these currents as free currents.

(ii) The macroscopic averaged bound charge and mass currents, which we identify as the sources of the magnetization and gravitomagnetization, appear in Eqs. (7) and (8) via the quantities $\mathcal{M}$ and $\mathcal{L}$, where $\mathcal{M}$ and $\mathcal{L}$ (Ref. 8) represent the macroscopic averaged magnetic dipole moment and angular momentum densities that appear in superconductors when external fields are present. Because the Cooper pairs possess no angular momentum and hence no magnetic dipole moment because of the fact that the electron pairs occupy the ground state with equal and opposite spin, the only possible contributors to $\mathcal{M}$ and $\mathcal{L}$ are the lattice ions such that

$$\mathbf{j}^s_{e,M}=\nabla\times\mathcal{M}=\frac{q_t\hbar}{2im_t}(\psi^*\nabla_t\psi-\psi\nabla_t\psi^*)\ , \tag{12}$$

$$\mathbf{j}^s_{m,M}=\nabla\times\mathcal{L}/2=\frac{\hbar}{2i}(\psi^*\nabla_t\psi-\psi\nabla_t\psi^*)\ , \tag{13}$$

where q_t and m_t are the charge and mass of a lattice ion, and ∇_t is only related to the coordinates of the lattice ions. It follows from the Le Chatelier principle[9] that the superconducting system should exert a stronger influence on the magnetic fields which aim to destroy it. The Cooper pairs reacting to the effect of a magnetic field less than B_c do not dissociate, but their center of gravity acquires considerable angular momentum, which gives rise to a superconducting condensate wave, and this induces a larger internal magnetic field. The interaction energy of this internal magnetic field with the magnetic moment of the lattice ions drives the lattice ions and superconducting condensate wave function to move together vortically within the range of the coherent length and results in an induced precession of the angular momentum of the lattice ions. Consequently, a time-dependent gravitomagnetic fields is generated within the superconductor until the condition $\mathbf{B}+(m/q)\mathbf{B}_g=\mathbf{0}$ is satisfied.

Then the total charge and mass currents given by Eqs. (5) and (6) can be replaced by

$$\mathbf{j}^s_e=2M_t|\psi|^2\nabla_t\phi-\frac{q^2}{m}\left[\mathbf{A}+\frac{m}{q}\mathbf{A}_g\right]|\psi|^2\ , \tag{14}$$

$$\mathbf{j}^s_m=2\frac{M_t}{\gamma_t}|\psi|^2\nabla_t\phi-q\left[\mathbf{A}+\frac{m}{q}\mathbf{A}_g\right]|\psi|^2\ , \tag{15}$$

where $M_t=q_t\hbar/2m_t$ is the Bohr magneton of a lattice ion and $\gamma_t=q_t/2m_t$ is its gyromagnetic ratio.

III. MAGNETIC ATTENUATION COEFFICIENT

Usually, the relative magnetic and gravitomagnetic permeabilities

$$\mu_r=1+\frac{\mathbf{j}^s_{e,M}\cdot\mathbf{j}^s_{e,f}}{(j^s_{e,f})^2} \tag{16}$$

and

$$\mu_{g,r}=1+\frac{\mathbf{j}^s_{m,M}\cdot\mathbf{j}^s_{m,f}}{(j^s_{m,f})^2} \tag{17}$$

are used to represent the effect of the bound currents, which are dependent on the material electromagnetic properties as well as on the internal structure of the superconductor. Then Eqs. (7) and (8) can be simply written in the form

$$\nabla\times\nabla\times\mathbf{A}=\mu\mathbf{j}^s_f-\mu_0\varepsilon_0\varepsilon_r\frac{\partial^2\mathbf{A}}{\partial t^2}\ , \tag{18}$$

$$\nabla\times\nabla\times\mathbf{A}_g=-\mu_g\mathbf{j}^s_{f,m}-\mu_{g,0}\varepsilon_{g,0}\frac{\partial^2\mathbf{A}_g}{\partial t^2}\ , \tag{19}$$

where the electric polarization $\mathbf{P}=\varepsilon_0\chi_e\mathbf{E}$ and the permittivity $\varepsilon=\varepsilon_0(\chi_e+1)=\varepsilon_0\varepsilon_r$ are used. Since there is no such of the gravitational counterpart of the electric dipole moment density $\mathbf{P}$, the necessary condition required by general relativity, $\varepsilon_g=\varepsilon_{g,0}$, is satisfied, i.e., the gravitoelectric permittivity $\varepsilon_g=1/4\pi G$ is a constant, where G is Newton's gravitational constant. The relationships of μ and μ_g with their free-space values μ_0 and $\mu_{g,0}$, respectively,

$$\mu=\mu_0\left[1+\frac{\mathbf{j}^s_{e,M}\cdot\mathbf{j}^s_{e,f}}{(j^s_{e,f})^2}\right]\ , \tag{20}$$

$$\mu_g = \mu_{g,0}\left[1+\frac{\mathbf{j}^s_{m,M}\cdot\mathbf{j}^s_{m,f}}{(j^s_{m,f})^2}\right], \tag{21}$$

follow at once. For understanding the physical meaning of Eqs. (20) and (21), we may consider the following cases.

(i) Clearly, if $\mathbf{j}^s_{e,M}$ and $\mathbf{j}^s_{m,M}$ are neglected, then Eqs. (20) and (21) assume their vacuum values, $\mu=\mu_0$ and $\mu_g=\mu_{g,0}$. Below, we demonstrate that the macroscopic quantum-mechanical nature of the superconducting state is crucial for understanding that these bound currents cannot be neglected. Equation (12) indicates that each lattice ion will acquire momentum from the interaction energy of its magneton with the magnetic field of the superconducting condensate wave function in the amount of

$$\mathbf{p}_t = 2\hbar\nabla_t\phi , \tag{22}$$

where the density of the lattice ions is assumed to be equal to that of the Cooper pairs as $|\psi|^2/2$ and the relation

$$\mathbf{j}^s_{e,M} = q_t|\psi|^2\mathbf{p}_t/2m_t \quad \text{or} \quad \mathbf{j}^s_{m,M} = |\psi|^2\mathbf{p}_t/2 \tag{23}$$

is used. Integrating Eq. (22) around a closed path, one finds[10]

$$\oint \mathbf{p}_t\cdot d\mathbf{l} = 2nh , \tag{24}$$

where the line integral of $\nabla_t\phi$ around a closed path must be set to an integral multiple n of 2π to satisfy the physical requirement that the order parameter ψ be single valued. One finds that the vortical momentum for a single superconducting lattice ion must be an integral multiple of Planck's constant h. The uncertainty principle provides a basis for arguing that the integral n is not zero. Since the product of momentum and position must satisfy the condition

$$xp_x \geq \frac{\hbar}{2} , \tag{25}$$

the quantum number n thus must assumes values

$$n \geq 1 . \tag{26}$$

It implies that each of the lattice ions will possess uniformly quantized angular momentum

$$\oint \mathcal{L}_t d\theta = 2nh \quad \text{or} \quad \mathcal{L}_t = 2n\hbar , \tag{27}$$

where $\mathcal{L}_t = \mathbf{r}\times\mathbf{p}_t$ and the integral element $d\mathbf{l}=\mathbf{r}\times(\mathcal{L}/\mathcal{L})d\theta$. Thus the angular momentum is quantized in integral multiples $\hbar$. We conclude that the uniform quantized vortical motion of the lattice ions results naturally in the fluxoid quantum of superconductivity regardless of whether or not there are holes in the order parameter ψ.

(ii) The perfect diamagnetic property of superconductors or the Meissner effect suggests that $\mu_r=0$. It is equivalent to the statement that the magnetic field is zero inside a superconductor. However, results of experimental measurements of the Meissner effect vary widely. Several theoretical models have been preoposed to explain the departures from the ideal Meissner effect, for example, the effect of trapped flux and impurities. To the best of our knowledge, the attenuation of magnetic fields by superconductors has as yet not been satisfactorily explained.

If the model presented in Ref. 1 is correct, then gravitational effects may be used to understand features of the Meissner effect, since the values of the ratio of μ_g and μ can be derived theoretically by rewriting Eqs. (20) and (21) as

$$\frac{\mu_{g,r}-1}{\mu_r-1} = \left[\frac{j^s_{e,f}}{j^s_{m,f}}\right]\left[\frac{j^s_{m,M}}{j^s_{e,M}}\right] . \tag{28}$$

Note that $\mathbf{j}^s_{e,f}\|\mathbf{j}^s_{m,f}$ and $\mathbf{j}^s_{m,M}\|\mathbf{j}^s_{e,M}$. Substituting Eqs. (9), (10), and (11)–(13) into Eq. (28) and taking account of the experimental fact that $\mu_r \ll 1$, we obtain

$$\frac{\mu_g}{\mu}\bigg/\frac{\mu_{g,0}}{\mu_0} = \frac{\mu_{g,r}}{\mu_r} \approx \frac{\gamma_c}{\gamma_t}\frac{1}{\mu_r} , \tag{29}$$

where $\gamma_c = |q|/2m$ is the gyromagnetic ratio for a Cooper pair. It should be pointed out that since nothing is known of the phase velocity v_p of a gravitational wave propagating within a superconductor, it is usually presumed to be equal to the velocity of light, c. We argue that the interaction of the coupled electromagnetic and gravitoelectromagnetic fields with the Cooper pairs in superconductors will form superconducting condensate waves characterized by a phase velocity v_p. Since $\mu_0\varepsilon_0 = 1/c^2 = \mu_{g,0}\varepsilon_{g,0}$ and $\mu\varepsilon = k^2/\omega^2 = \mu_g\varepsilon_g = v_p^{-2}$, the phase velocity can be predicted for the first time as

$$v_p = \frac{\omega}{k} = \frac{c}{\sqrt{\mu_{g,r}}} \sim 10^6\,(\mathrm{m/s}) , \tag{30}$$

which is 2 orders of magnitude smaller than the velocity of light, c. This difference arises because $\varepsilon_g=\varepsilon_{g,0}$ and the value of the gravitomagnetic permeability $\mu_g = (\gamma_c/\gamma_t)\mu_{g,0}$ given in Eq. (29) is γ_c/γ_t orders of the magnitude larger than its free-space value $\mu_{g,0}\sim 10^{-26}$.

Finally, one finds that the magnetic attenuation coefficient

$$\beta \approx \frac{m^2\mu_{g,0}}{q^2\mu_0}\frac{\gamma_c}{\gamma_t}\frac{1}{\mu_r} \tag{31}$$

is $(\gamma_c/\gamma_t)(1/\mu_r)$ times larger than its free-space value of $\beta_0 = m^2\mu_{g,0}/q^2\mu_0 \sim 10^{-43}$ and its magnitude is determined mainly by the magnetic permeability of the superconductor. For example, if Shiff's screening factor[11] $\beta\sim 10^{-7}$ and $\gamma_t = 3.24\times 10^7\ \mathrm{s^{-1}\,T^{-1}}$ for ^3H_e are used in Eq. (31), then one finds that the permeability of ^3H_e is the order of $\mu_r \sim 10^{-33}$, which is small enough to have a very significant consequences for the internal magnetic field. Our finding is not only supported by Schiff's theoretical screening factor, but also experimentally supported by the measurement of minimum remnant magnetic fields of $\mathbf{B}\sim 10^{-12}$ T,[4] which corresponds to attenuation of the Earth's magnetic field by a factor of $\sim 10^{-7}$.

IV. σ-μ MODEL

The measurement of μ for superconductors is difficult, because its values are near zero. Thus a theoretical model that relates μ to some measurable quantity is needed. Below, we develop a model which functionally relates μ with the conductivity σ. Choosing the London gauge for the vector potentials

$$(\nabla\cdot\mathbf{A})=0\ ,\quad (\nabla\cdot\mathbf{A}_g)=0 \tag{32}$$

and introducing the Fourier components of the appropriate quantities in Eqs. (9), (18), and (19), we obtain

$$\mathbf{j}_e^s(\mathbf{k},\omega)=-Q(\mathbf{k})\left\{\mathbf{A}(\mathbf{k},\omega)+\frac{m}{q}\mathbf{A}_g(\mathbf{k},\omega)\right\}, \tag{33}$$

$$k^2\left[\frac{\mu_r-1}{\mu_r}\right]\mathbf{A}(\mathbf{k},\omega)=-\mu Q(\mathbf{k})\left[\mathbf{A}(\mathbf{k},\omega)+\frac{m}{q}\mathbf{A}_g(\mathbf{k},\omega)\right], \tag{34}$$

$$k^2\left[\frac{\mu_{g,r}-1}{\mu_{g,r}}\right]\mathbf{A}_g(k,\omega)=\mu_g Q(\mathbf{k})\frac{m}{q}\left[\mathbf{A}(\mathbf{k},\omega)+\frac{m}{q}\mathbf{A}_g(\mathbf{k},\omega)\right], \tag{35}$$

$$\mathbf{k}\cdot\mathbf{A}(\mathbf{k},\omega)=0\ ,\quad \mathbf{k}\cdot\mathbf{A}_g(\mathbf{k},\omega)=0\ , \tag{36}$$

which yields the result

$$\frac{\mathbf{A}_g}{\mathbf{A}}=-\frac{m\mu_g}{q\mu}\frac{\mu_r-1}{\mu_r}\frac{\mu_{g,r}}{\mu_{g,r}-1}\ . \tag{37}$$

Equating two expressions for the current,

$$\mathbf{j}_e^s(\mathbf{k},\omega)=\sigma(\mathbf{k},\omega)\mathbf{E}(\mathbf{k},\omega)=-Q(\mathbf{k})\left[1-\frac{m^2\mu_g}{q^2\mu}\frac{\mu_r-1}{\mu_r}\frac{\mu_{g,r}}{\mu_{g,r}-1}\right]\mathbf{A}(\mathbf{k},\omega)\ , \tag{38}$$

we can define a complex conductivity proportional to $Q(\mathbf{k})$ such that

$$\sigma(\mathbf{k},\omega)=\frac{-iQ(\mathbf{k})}{\omega}\left[1-\frac{m^2\mu_g}{q^2\mu}\frac{\mu_r-1}{\mu_r}\frac{\mu_{g,r}}{\mu_{g,r}-1}\right]. \tag{39}$$

Before we examine further the relationship between μ_r and σ, we consider the expression for the Kernel function given in Eq. (11). One can see that the kernal function is independent of $\mathbf{k}$. Thus the complex conductivity [Eq. (39)] can be written as a function of ω only of the form $\sigma(\omega)=\sigma_1(\omega)-i\sigma_2(\omega)$. Both the real and imaginary parts of $\sigma(\omega)$ enter into the determination of the response of a superconductor to time-dependent electromagnetic and gravitoelectromagnetic fields, as shown in Fig. 1.[7] For a given E field, the energy absorption per unit volume is determined by the real part, i.e., $\sigma_1(\omega)E^2$. One sees from Fig. 1 that at low temperature $\sigma_1(\omega)$ falls exponentially, and for $\hbar\omega/2\Delta\le 1$ ($\Delta=1.73\hbar T_c$), $\sigma_1(\omega)$ is

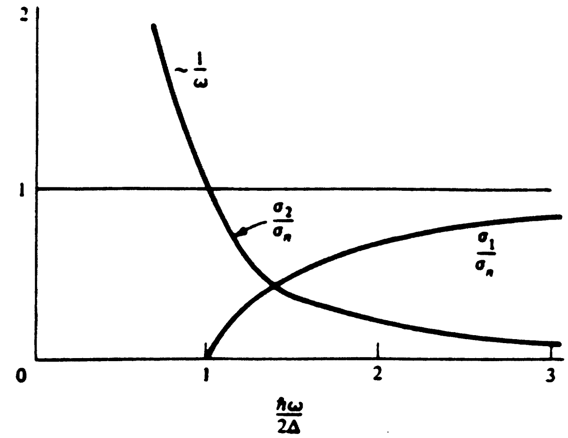

FIG. 1. Complex conductivity of superconductors in the extreme anomalous limit (or extreme dirty limit) at $T=0$. The rise of σ_2 as $1/\omega$ below the gap describes the accelerative supercurrent response (Ref. 7).

equal to zero. Of particular importance is the proportionality of $\sigma_2(\omega)$ to $1/\omega$, as shown in Fig. 1. One sees that $T\to 0$ as $\omega\to 0$ and $\sigma_2(\omega)\propto 1/\omega\to\infty$. In fact, in the range of $\hbar\omega/2\Delta\le 1$ there are no thermal excitation mechanisms present, and the only process allowing absorption of energy is the creation of the Cooper pairs, which is determined by only $\sigma_2(\omega)$. This allows one to drop the real part and to write the conductivity as

$$\sigma(\omega)=\frac{1}{\omega}\frac{\omega_{\rm pl}^2}{4\pi}\left[1-\frac{m^2\mu_g}{q^2\mu}\frac{\mu_r-1}{\mu_r}\frac{\mu_{g,r}}{\mu_{g,r}-1}\right], \tag{40}$$

where $\omega_{\rm pl}^2=4\pi n^s q^2/m$ is the plasma frequency. One may write Eq. (40) in the standard form for the temperature-dependent conductivity,[12] namely,

$$\sigma=\frac{\omega_{\rm pl}^2}{4\pi}\tau\left[1+\frac{q^2 m_t\mu_0}{m^2\mu_{g,0}}\frac{\gamma_t}{\gamma_c}\beta^2\right], \tag{41}$$

obtained by integrating $\sigma(\omega)$ in Eq. (40) over frequency space, where the expression for τ is given by[12]

$$\frac{\hbar}{\tau}=4\pi k_b T\int_0^\infty\frac{d\omega}{\omega}\alpha^2(\omega)F(\omega)\left[\frac{x}{\sinh x}\right]^2, \tag{42}$$

with $x=\hbar\omega/2k_bT$. The expansion of $x/\sinh x$ for $x\le 1$ is

$$\frac{x}{\sinh x}\approx\frac{1}{1+x^2/6+x^4/120+\cdots}\approx 1-\frac{x^2}{6}-\frac{x^4}{120}-\cdots\ . \tag{43}$$

It yields the coupling constant

$$\lambda^*=2\int_0^\infty\frac{d\omega}{\omega}\alpha^2(\omega)F(\omega)[1-O(x^2)]\ . \tag{44}$$

Substituting Eqs. (42) and (44) into Eq. (41), it follows at once that

$$\sigma(T)=\frac{\omega_{\rm pl}^2}{4\pi}\frac{\hbar}{2\pi k_b T\lambda^*}\left[1+\frac{q^2 m_t \mu_0}{m^2\mu_{g,0}}\frac{\gamma_t}{\gamma_c}\beta^2\right]. \tag{45}$$

If the gravitational effect is neglected, i.e., if there is complete shielding, i.e., $\beta=0$. Equation (45) then becomes the well-known formula for the conductivity,

$$\sigma_0=\frac{n^s q^2}{m}\tau_0\,, \tag{46}$$

where the relaxation time

$$\tau_0=\frac{\hbar}{2\pi k_b T\lambda^*} \tag{47}$$

describes the time constant for the supercurrent to die away when the applied fields are moved. Its value is determined by the temperature T and the coupling constant λ^*. In copper, for example, $n^s=8.5\times10^{28}\ \mathrm{m}^{-3}$, at $T=273$ K, the conductivity $\sigma_0=6.4\times10^7\Omega^{-1}\mathrm{m}^{-1}$, and hence the relaxation time $\tau=2.68\times10^{-14}$ s. However, at $T=4$ K, copper has a conductivity 606 times higher, so that the relaxation times becomes $\tau_0=1.62\times10^{-11}$ s. This result would appear to indicate that the copper is not a superconductor even at very low temperatures. Thus, except for the condition of low temperature, superconductors must be characterized by some other unique properties which distinguish them from normal metals at low temperature which allows the near-infinite conductivity.

Equation (45) suggests that one of these unique properties is the near-zero permeability. Rewriting the conductivity approximately as

$$\sigma\approx\sigma_0\frac{m^2\mu_{g,0}}{q^2\mu_0}\frac{\gamma_t}{\gamma_c}\frac{1}{\mu_r^2}\,, \tag{48}$$

one sees that $\mu_r\to0$, $\sigma\propto1/\mu_r^2\to\infty$. Estimating the order of magnitude of the terms in the above equation, $m^2\mu_{g,0}/q^2\mu_0\sim O(10^{-43})$, $\gamma_c/\gamma_t\sim O(10^4)$, and $\sigma_0\sim O(10^{10})$, one finds that

$$\sigma\sim10^{-29}\frac{1}{\mu_r^2}\,. \tag{49}$$

Substituting published measurements of the conductivities[13] (which are at least larger than $2.4\times10^{24}\ \Omega^{-1}\mathrm{m}^{-1}$ for superconductors) into Eq. (49), one finds that the experimental measurement of σ indicates that $\mu_r\sim O(10^{-27})$, which is of the order of 10^5 larger than the theoretical value obtained by using Shiff screening factor for ^3H_e.

There exists, however, an inconsistency of the relaxation time as derived from conventional superconductivity theory. Equation (48) also can be simply written in the form of the relaxation time

$$\tau\approx\tau_0\frac{m^2\mu_{g,0}}{q^2\mu_0}\frac{\gamma_t}{\gamma_c}\frac{1}{\mu_r^2}\,. \tag{50}$$

If $\tau_0=\hbar/2\pi k_b T\lambda^*\sim O(10^{-11})$ and if $\mu_r\sim O(10^{-27})$, Eq. (50) gives $\tau\sim O(10^4)$ s. Although this result is of order 10^{15} longer than that of copper at low temperatures, it is still not consistent with the extremely long persistence time of supercurrents. Previously proposed resolutions of this apparent inconsistency are that at any nonzero frequency there will be an ac noise voltage reflecting the real part of the ac impedance of the superconductor. Then a superconductor is really a perfect conductor only for direct currents, since, as $\omega\to0$, $\sigma\to\infty$. We argue that this proposed resolution is not an experimental fact.

We suggest that this inconsistency may be caused by taking integral of Eq. (40) over all frequency space. Since the frequency ω is related to the temperature, when integrated over the whole of frequency space, the result actually includes the nonsuperconducting range of conductivity as well. Therefore an additional equation is required in order to limit the temperature effect as well as to isolate the magnetic permeability effect. In fact, if $\mathcal{M}$ and $\mathcal{L}$ are explicitly included as in Eqs. (7) and (8), Eqs. (33)–(35) give

$$\mathbf{A}(\mathbf{k},\omega)+\frac{m}{q}\mathbf{A}_g(\mathbf{k},\omega)=\frac{i\mathbf{k}\times\dfrac{\mu}{\mu_r-1}\mathcal{M}(\mathbf{k},\omega)-i\mathbf{k}\times\dfrac{\mu_g}{\mu_{g,r}-1}\dfrac{m}{2q}\mathcal{L}(\mathbf{k},\omega)}{k^2+Q(\mathbf{k})\dfrac{\mu}{\mu^r-1}\left[1-\dfrac{\mu_{g,0}m^2}{\mu_0 q^2}\dfrac{\mu_r-1}{\mu_r}\dfrac{\mu_{g,r}}{\mu_{g,r}-1}\right]}\,, \tag{51}$$

$$\mathbf{j}_e^s(\mathbf{k},\omega)=-Q(\mathbf{k})\frac{i\mathbf{k}\times\mu_0\mathcal{M}(\mathbf{k},\omega)\dfrac{\mu_r}{\mu_r-1}-i\mathbf{k}\times\dfrac{m}{2q}\mu_{g,0}\mathcal{L}(\mathbf{k},\omega)\dfrac{\mu_{g,r}}{\mu_{g,r}-1}}{k^2+\mu_0 Q(\mathbf{k})\dfrac{\mu_r}{\mu_r-1}\left[1-\dfrac{\mu_{g,0}m^2}{\mu_0 q^2}\dfrac{\mu_r-1}{\mu_r}\dfrac{\mu_{g,r}}{\mu_{g,r}-1}\right]}\,. \tag{52}$$

The above two equations yield the needed additional equation

$$\varepsilon\omega^2=-Q(\mathbf{k})\frac{\mu_r}{\mu_r-1}\left[1-\frac{m^2\mu_g}{q^2\mu}\frac{\mu_r-1}{\mu_r}\frac{\mu_{g,r}}{\mu_{g,r}-1}\right]. \tag{53}$$

Combining Eq. (53) with Eq. (39) to eliminate any explicit dependence on ω, the major role played by the gravitational effect in superconductivities becomes apparent, since we find the following simple relation between σ and μ, which we refer to as the σ-μ model:

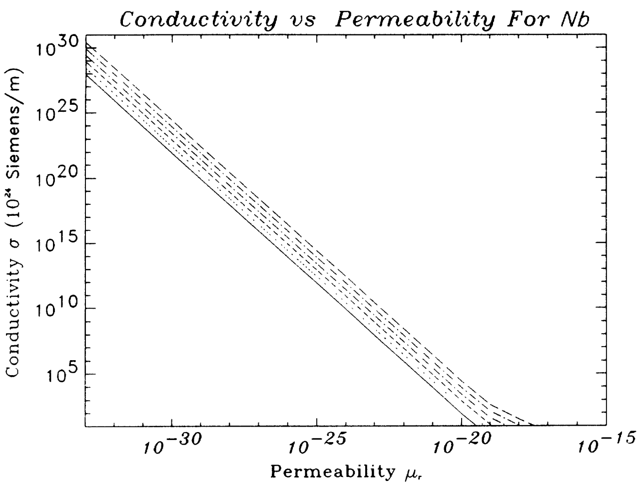

FIG. 2. σ-μ model: conductivity as a function of permeability computed from Eq. (54) for several value of density of the Cooper pair of the order from 10^{23} m^{-3} (bottom line) to 10^{29} m^{-3} (top line) for niobium.

$$\sigma^2=\frac{m_t\varepsilon_0 Q}{m}\left[\frac{mm_t\mu_{g,0}}{q^2\mu_0}\frac{1}{\mu_r^2}+1\right]\frac{1}{\mu_r^2}\ . \tag{54}$$

One sees that the conductivity is only determined by the magnetic permeability or vice versa. The conductivity calculated from Eq. (54) as a function of permeability and the density of the Cooper pairs for Nb is illustrated by the numerically computed logarithmic curves in Fig. 2. It can be seen from Fig. 2 that the conductivity increases exponentially with continuously decreasing permeability. This characteristic dependence can be used to obtain the values of μ experimentally. For example, Fig. 2 shows that the experimentally measured values of conductivities larger than the order of 10^{25} are consistent with permeabilities smaller than 10^{-20} over densities ranging from 10^{23} to 10^{29} m^{-3} for a Nb superconductor.

The implications of the σ-μ model are made clear by returning to consideration of the relaxation time introduced in Eq. (46). Equation (54) can be written in the alternative form

$$\tau=\frac{1}{\mu_r}\left[\frac{m_t\varepsilon_0}{mQ}\left[\frac{mm_t\mu_{g,0}}{q^2\mu_0}\frac{1}{\mu_r^2}+1\right]\right]^{1/2}\ . \tag{55}$$

By varying μ_r from 10^{-33} to 10^{-20}, the numerically computed curves of relaxation time as a function of permeability based on Eq. (55) are those illustrated in Fig. 3. It is shown that the relaxation time increases exponentially, and for the typical case of $\mu_r < 10^{-20}$, the supercurrent can persist for at least years. This extremely long lifetime for the supercurrent is indeed consistent with the experimental observations of the persistence of superconductivity.

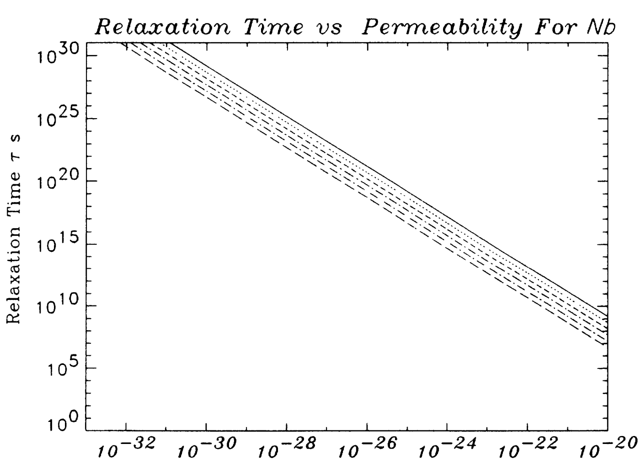

FIG. 3. Relaxation time dependence of permeability for various densities of the order from 10^{23} m^{-3} (bottom line) to 10^{29} m^{-3} (top line) for niobium calculated based on Eq. (55).

V. CONCLUSIONS

Expressing the conductivity in Eq. (54) as a function of the magnetic attenuation coefficient as shown in Fig. 4, one sees that for typical observed conductivities larger than the order of 10^{25}, the corresponding magnetic attenuation coefficient is larger than the order of 10^{-20}. Comparing this experimental limit with the extreme theoretical approximation of Schiff's result, a possible range for the magnetic attenuation coefficient can be predicted as

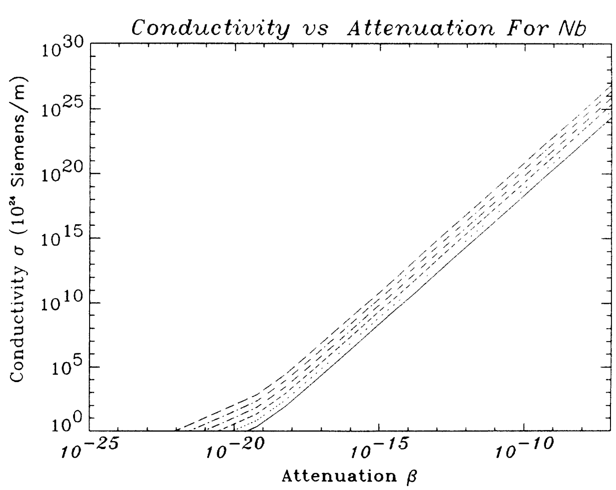

FIG. 4. Conductivity as a function of the magnetic attenuation coefficient calculated from Eq. (55) for different densities of the order from 10^{23} m^{-3} (bottom line) to 10^{29} m^{-3} (top line) for niobium.

$$10^{-7} > \beta > 10^{-20} . \tag{56}$$

The result suggests that the experimentally observed very poor Meissner effect[4] may not be simply caused by impurities alone, but may be the result of a more fundamental physical mechanism, namely, gravitational effects.

ACKNOWLEDGMENTS

We are greatly indebted to Dr. C. A. Lundquist, Dr. C. M. Will, and Dr. Jeeva Anandan for valuable discussions on this work.

[1]B. Mashhoon, H. J. Paik, and C. M. Will, Phys. Rev. D **39**, 2825 (1989).

[2]J. P. Turneaure *et al.*, Adv. Space Res. **9**, 29 (1989).

[3]C. W. F. Everitt, in *Near Zero: New Frontiers of Physics,* edited by J. D. Fairbank, B. C. Deaver, Jr., C. W. F. Everitt, and P. F. Michelson (Freeman, New York, 1988), p. 587.

[4]B. Cabrera, in *Near Zero: New Frontiers of Physics* (Ref. 3), p. 312.

[5]N. Li and D. G. Torr, Phys. Rev. D **43**, 457 (1990). In contrast with the Meissner effect, Eq. (31) in Ref. 5 indicates that a residual magnetic field as a function of $\mathbf{B}(z) \approx (-m^2\mu_g/q^2\mu)\mathbf{B}_0$ exists within a superconductor and its relationship with the external magnetic field $\mathbf{B}_0$ is determined by the magnetic attenuation coefficient β defined by $\beta = m^2\mu_g/q^2\mu$ as rewritten in Eq. (1) in this paper.

[6]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, New York, 1976), p. 455.

[7]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975), pp. 60 and 70.

[8]K. S. Thorne, in *Near Zero: New Frontiers of Physics* (Ref. 3), p. 573.

[9]A. A. Matsutsin, Fiz. Tverd. Tela (Leningrad) **30**, 2683 (1988) [Sov. Phys. Solid State **30**, 1546 (1988)].

[10]C. N. Yang, in *Near Zero: New Frontiers of Physics* (Ref. 3), p. 253.

[11]L. I. Schiff, Phys. Rev. **132**, 2194 (1963).

[12]B. Chakrakorty, W. E. Pickett, and P. B. Allen, Phys. Rev. B **14**, 3227 (1976).

[13]R. C. Weast and M. J. Astle, *CRC Handbook of Chemistry and Physics,* 59th ed. (CRC, Boca Raton, FL, 1979), p. E-85.